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Induction - hints

1 Exercises

Beginner

1.1 Show the difference is always even. What does this imply?

1.2 Draw a diagonal and look at the two smaller polygons.

1.3 When going from n to $n+1$, remember that your induction hypothesis allows you to interchange both sides of the statement!

1.4 From when onwards is the inequality no longer satisfiable? How do we go from $n!$ to $(n+1)!$? And how do we go from 3^n to 3^{n+1}?

1.5 Take the inequality for n, add $\frac{1}{(n+1)^2}$ on both sides and find a common denominator for the right hand side. Is it possible to remove something from the fraction in order to simplify the inequality (while also keeping it correct)?

1.6 Write the $n+1$ term as a sum of terms all divisible by 47.

Advanced

1.7 Use strong induction and consider the n even / n odd cases.

1.8 Find a formula to calculate a number based on the previous one.

1.9 Consider the $(n+1)$th line. How many times does it intersect with the remaining n lines? What does that imply about the number of new regions?

1.10 Where is it possible to place the tallest block?

1.11 Instead of adding a new point, remove one in order to minimise the number of edges removed.

1.12 There are two cases: whether the last element is in the subset. No need to find an explicit formula, just google "Fibonacci".

1.13 Consider an extra person at the end of the line.

1.14 How many steps are needed to get the largest book on platform 3 ? Find a formula for that and prove it.

Olympiad

1.15 Try to decompose a valid decomposition into more squares. Several base cases might be necessary.

1.16 On the $2^{n+1} \times 2^{n+1}$ chessboard, consider the quarter with the missing square. Is it possible to place a L-triomino such that the inductive hypothesis can be applied to the remaining three quarters?

1.17 Instead of adding a car, remove the right car and add its oil to a remaining car. What can we conclude about the case with $n + 1$ cars thanks to the case with n cars?

1.18 Find Bob's winning strategy its the pattern. Prove that pattern.