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IMO Selection 2026

First Exam

Bern

May 9, 2026

Duration: 4.5 hours

Difficulty: The problems are ordered by difficulty.

Points: Each problem is worth 7 points.

1. Lord Shen and Po are playing a game on a regular 2026-gon. Lord Shen writes a positive real number in red at each vertex, such that the product of the 2026 red numbers (which are not necessarily distinct) is 1.

Then, Po draws non-intersecting line segments between the vertices of the 2026-gon, in a way that each created region is a triangle. Inside each of these triangles, he writes out the product of the three red numbers written on its vertices in green.

Determine the largest real number c such that, no matter the numbers chosen by Lord Shen, Po can make sure that the sum of the green numbers is at least c.

2. Let A, B, X, C be points on a circle ω in this order, such that $AX > AB, AC$. Let $P, Q \neq X$ be points on lines XB and XC respectively, such that $AP = AX = AQ$. Let M and N denote the midpoints of BC and PQ respectively. If $S \neq A$ is the second intersection of NA with ω, prove that $MN = MS$.

3. Heidi is placing Ricola on an empty 2026×2026 grid. At each step, Heidi chooses a row or column in which the total number of Ricola is a multiple of 3, chooses an empty cell in this row or column, and places a single Ricola in it. If there is no such row or column, Heidi stops.

Determine the maximum number of Ricola that Heidi can place.

Good luck!

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Second Exam

Bern

May 10, 2026

Duration: 4.5 hours

Difficulty: The problems are ordered by difficulty.

Points: Each problem is worth 7 points.

4. There is a row of $n \geq 3$ icebergs, labelled from left to right with the integers 1 to n. Pingu starts dancing on one of the icebergs and then moves according to the following rules.

If Pingu has just danced on iceberg k, he then makes a sequence of k hops to an adjacent iceberg. The first of the k hops is always to the right, unless Pingu is one iceberg n, in which case it is to the left. Each of the subsequent $k-1$ hops is in the same direction as the previous hop, unless Pingu is on iceberg 1 or n in which case he changes direction. Pingu dances on the iceberg he lands on after k hops.

Pingu continues hopping and dancing in this manner indefinitely. An iceberg is *inevitable* if Pingu eventually dances on it, regardless of the iceberg he starts on. Determine all $n \geq 3$ for which there is exactly one inevitable iceberg.

Example: If $n = 8$, and Pingu first dances on iceberg 3, then the next four icebergs he dances on are $6, 4, 8$ and 2, in this order.

5. Determine all functions $f : \mathbb{N} \to \mathbb{Z}$ with the following property: for every positive integer n and every positive divisor a of n there exists a positive divisor b of n such that

$$n \mid a + f(b).$$

6. Determine all functions $f : \mathbb{R} \to \mathbb{R}$ such that for all $x, y \in \mathbb{R}$,

$$f(f(y) - f(x)) + 4f(xy) = f(f(x) + y^2).$$

Good luck!

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Third Exam

Bern

May 23, 2026

Duration: 4.5 hours
Difficulty: The problems are ordered by difficulty.
Points: Each problem is worth 7 points.

7. Let n be a positive integer. Determine all sequences of integers $1 \leq a_1 \leq \cdots \leq a_n$ such that

- each a_i is a power of two, and
- $(2^{a_1}+1)(2^{a_2}+1)\cdots(2^{a_n}+1) = 2^{1+a_1+\cdots+a_n} - 1$.

8. Let n be a positive integer. There is a magical door at Hogwarts with n levers labelled from 1 to n. Lever i can either be *up* or *down*, and toggling between the two costs 2^{i-1} Galleons. Every possible combination of lever positions makes the door lead to a different location. Initially, all the levers are down.

The 2^n members of Dumbledore's Army are being chased by Voldemort and want to use the magical door to split up so that each person travels to a different location. What is the minimum amount of Galleons they have to spend in total?

Remark: Dumbledore's Army may send a member through the door before switching any lever.

9. Let ABC be a triangle with circumcircle Ω. The tangent to Ω at A is called t. Let ω_B be the circle outside of ABC, tangent to t and to AB at B. Similarly, let ω_C be the circle outside of ABC, tangent to t and to AC at C. Lastly, let P be the intersection of the two common internal tangents of ω_B and ω_C, and let Γ denote the circumcircle of the triangle PBC.

If the common tangents to Ω and Γ intersect at a point T, prove that A, P and T are collinear.

Remark: A common internal tangent ℓ to two circles is tangent to both of them, in such a way that the two circles ***are not*** *on the same side of ℓ.*

Good luck!

IMO Selection 2026

Fourth Exam

Bern

May 24, 2026

Duration: 4.5 hours

Difficulty: The problems are ordered by difficulty.

Points: Each problem is worth 7 points.

10. Let ABC be an acute triangle. Let I_B and I_C be the excenters of triangle ABC opposite B and C, respectively. Let E be a point on line BA such that $\angle EI_BB = 90°$. Let F be a point on line CA such that $\angle CI_CF = 90°$. Let the circle with center I_B and radius I_BE intersect the segment I_BB at X. Similarly, let the circle with center I_C and radius I_CF intersect the segment I_CC at Y.

Prove that BY is perpendicular to CX.

Remark: The excenter of a triangle ABC opposite vertex B is the center of the circle that is tangent to line segment AC, to ray BA beyond A, and to ray BC beyond C. The excenter opposite C is similarly defined.

11. Let $f\colon \mathbb{Z}_{\geq 0} \to \mathbb{Z}_{\geq 0}$ be a function **that takes on infinitely many values**. Assume that for all non-negative integers m and n that satisfy

$$f(m+n) = \max\Big\{f(0),\ f(1),\ \ldots,\ f(m+n)\Big\},$$

we have $f(m+n) = f(m) + f(n)$.

Prove that there exist positive integers A, B, C and D such that for all non-negative n we have $f(An+B) = Cn + D$.

12. For positive integers n, k, we define $0 \leq f_k(n) < 2^k - 1$ to be the remainder of the division of n by $2^k - 1$. We say that a positive integer n is a *panda* if $f_k(n) \leq f_{k+1}(n)$ for every positive integer k.

(a) Prove that there exist infinitely many pandas.

(b) Prove that there exists a positive integer M such that the number of pandas less than or equal to M is at most $\frac{M}{2026^{2026}}$.

Good luck!