MATHEMATICAL.
OLYMPIAD.CH
MATHEMATIK-OLYMPIADE
OLYMPIADES DE MATHÉMATIQUES
OLIMPIADI DELLA MATEMATICA

IMO Selection 2025

First Exam

Bern

May 10, 2025

Duration: 4.5 hours
Difficulty: The problems are ordered by difficulty.
Points: Each problem is worth 7 points.

1. Let k be a positive integer. Leo owns an empty garden that measures 45 by 45 units. He wants to plant flowers in each of the 2025 unit squares, and can plant one square with flowers each morning. However, whenever flowers are planted in a square, then k days later in the evening, any orthogonally adjacent empty square will become infested with weeds. Infested squares can no longer have flowers planted in them.

Find the minimum value of k such that Leo can plant his entire garden with flowers.

2. Let n be a positive integer and let $P(x)$ denote the polynomial

$$x^n + a_{n-1}x^{n-1} + \cdots + a_1x + a_0$$

with real coefficients and $a_0 \neq 0$, whose n roots are all pairwise distinct, positive real numbers. Assuming that $P(x)$ divides $P(2x)P(x/2)$, prove that

$$\frac{a_{n-1}a_1}{a_0} \geq \frac{9n^2}{8}.$$

3. Let Ω be a circle and ω a circle inside Ω. Point A lies on Ω, and tangents to ω through A touch ω at B and C. Let line BC intersect Ω at points X and Y. Let K, L and M be the midpoints of BC, AX and AY respectively. The circumcircle of XLK intersects ω at two points P_1 and P_2, such that P_1 lies on the same side of BC as A. Similarly, the circumcircle of YMK intersects ω at two points Q_1 and Q_2, such that Q_1 lies on the same side of BC as A. If lines P_1Q_1 and P_2Q_2 intersect at R, prove that RA is tangent to Ω.

Good luck!

IMO Selection 2025

Second Exam

Bern

May 11, 2025

Duration: 4.5 hours

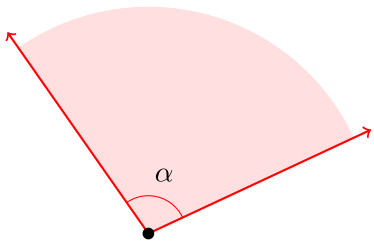
Difficulty: The problems are ordered by difficulty.

Points: Each problem is worth 7 points.

4. Let ABC be a triangle with $AB < AC$. Denote its circumcircle by Ω, its circumcenter by O, and its incenter by I. Let M be the midpoint of the arc BC not containing A. Line OI meets Ω at E and F, and line BC meets ME and MF at K and L, respectively. Suppose that $IA = IM$. Prove that $IKML$ is a rectangle.

5. Let α be a real number satisfying $0 < \alpha < 180$. For Leo's birthday, Frieder has placed 2025 gnomes at arbitrary points inside his garden. No three gnomes are collinear and no two gnomes coincide. Each gnome has a field of view spanning α degrees (including the boundary). After Frieder places the gnomes down, Leo wants to rotate the gnomes such that, for each gnome, the number of other gnomes it sees is different.

Determine all values of α for which Leo can achieve this, regardless of how the gnomes are placed.



An example of the field of view of a gnome. It extends infinitely between the boundary rays.

6. Let $a_1, a_2, a_3, \ldots$ be an infinite sequence of real numbers satisfying, for all integers $n \geq 1$,

$$a_{\lfloor \frac{n}{1} \rfloor} \cdot a_{\lfloor \frac{n}{2} \rfloor} \cdot \ldots \cdot a_{\lfloor \frac{n}{n} \rfloor} = 2^{n^2}.$$

Prove that $\frac{a_{n+1}-a_n}{n+1}$ is an integer for all $n \geq 1$.

Good luck!

MATHEMATICAL.
OLYMPIAD.CH
MATHEMATIK-OLYMPIADE
OLYMPIADES DE MATHÉMATIQUES
OLIMPIADI DELLA MATEMATICA

IMO Selection 2025

Third Exam

Bern

May 24, 2025

Duration: 4.5 hours
Difficulty: The problems are ordered by difficulty.
Points: Each problem is worth 7 points.

7. Determine all functions $f : \mathbb{R} \to \mathbb{R}$ such that

$$f(xf(y) - yf(x)) = f(xy) - xy$$

holds for all $x, y \in \mathbb{R}$.

8. Let a and b be integers with $a > b \geq 5$. Prove that there exist a positive integer k and positive integers $c_1, c_2, \ldots, c_k$ with $c_1 = a$ and $c_k = b$, such that $c_i^2 + c_{i+1}^2$ is a perfect square for all $1 \leq i < k$.

9. Leo's garden has now been passed on to his niece Batlpàsz, who is a bit disappointed by her inheritance at first. However, one day, she gets a letter from Frieder telling her that there is buried treasure under one of the 2025 gnomes in the garden, but she doesn't know which one.

Some pairs of gnomes are directly connected by a dirt path, and at most three dirt paths meet at each gnome. Bàtlpasz can get from any gnome to any other gnome by walking along some of the dirt paths. However, removing any single dirt path would split the gnomes into exactly two separate connected groups, with no way to get from one group to the other.

In the middle of each dirt path is a fairy that can tell Batlpàsz which direction along the path she has to go to get to the gnome with the treasure. Every morning for a fortnight, Batlpàsz may ask a fairy for help. After the fourteenth question, if Batlpàsz has not found the location of the treasure, it will vanish. Can Batlpàsz always find the treasure, no matter how the dirt paths are arranged?

Good luck!

IMO Selection 2025

Fourth Exam

Bern

May 25, 2025

Duration: 4.5 hours

Difficulty: The problems are ordered by difficulty.

Points: Each problem is worth 7 points.

10. Determine all $n \in \mathbb{N}$ with at least four positive divisors satisfying the following property:
For any distinct positive divisors $a, b \notin \{1, n\}$, we have $\gcd(a^b + 1, n) > 1$.

11. Let Γ be a fixed circle with two fixed points A, B on it. For a variable point $P \notin \{A, B\}$ on Γ, let G be the center of gravity of the triangle ABP. The parallel to line GP through B intersects AP at point C, and the line CG intersects the circumcircle of the triangle ABG a second time at point Q. Prove that as P varies, the point Q lies on a fixed circle.

12. Find all polynomials $P \in \mathbb{Z}[x]$ for which there exists a non-constant polynomial $Q \in \mathbb{Z}[x]$ satisfying the following:
For all integers a, b we have $P(b - a) \mid Q(b) - Q(a)$.

Good luck!