Second Round 2025

MATHEMATICAL. OLYMPIAD.CH
MATHEMATIK-OLYMPIADE
OLYMPIADES DE MATHÉMATIQUES
OLIMPIADI DELLA MATEMATICA

Zürich, Lausanne, Lugano
December 21, 2024

Duration: 3 hours
Difficulty: The problems of each topic are ordered by difficulty.
Points: Each problem is worth 7 points.

Geometry

G1) Let ABC be a triangle with circumcircle Ω, and k be a circle passing through B and C such that A is in the interior of k. The tangent to Ω through A intersects k at two points P and Q, such that P and C are on different sides of AB. If M is the intersection of AB and PC and N is the intersection of AC and QB, prove that MN is parallel to PQ.

G2) Let ABC be a triangle with $AC > BC$. Its incircle touches sides BC, CA and AB at D, E and F, respectively. Let P be the point on segment AC such that $BP \parallel DE$. Let Ω be the circumcircle of triangle AFD. Line EF intersects Ω again at Q and line PQ intersects Ω again at R. Prove that $PEBR$ is a cyclic quadrilateral.

Combinatorics

C1) Let n be a positive integer. Pingu the penguin and his n penguin friends are collecting salmon. Every penguin has at most n salmon, and no two penguins have the same number of salmon. In how many ways can the $n+1$ penguins split into some number of groups of arbitrary sizes, such that each group has exactly n total salmon?

C2) The n participants of the Olympiad are trying to escape Wonderland. They arrive at a row of n closed doors, ordered in decreasing size. For every $1 \leq k \leq n$, there is exactly one participant that fits through the first k doors, but not the others. One by one, in some order, the participants approach the row of doors to pass through it. Each participant walks along the row of doors, starting at the largest one. If they come across an open door they fit through, they walk through it and close it behind them. If they reach the last door they fit through and it is closed, they open it and go through it, leaving it open behind them. If at the end all doors are closed again, all the participants can escape safely. In how many different orders can the n participants approach the row of doors while achieving this?

Number Theory

N1) Let a, b be positive integers. Prove that the expression

$$\frac{\gcd(a+b, ab)}{\gcd(a, b)}$$

is always a positive integer, and determine all possible values it can take.

N2) Determine all triples (a, b, p) of positive integers where p is a prime number and

$$p(a+b) = a^2(2p^2 - pb + 1).$$

Good luck!