Second Round 2024

MATHEMATICAL.
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Zürich
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Duration: 3 hours
Difficulty: The problems of each topic are ordered by difficulty.
Points: Each problem is worth 7 points.

Geometry

G1) Let ABC be a triangle. The angle bisector of $\angle ACB$ intersects AB in D. Let T and H be points on the circumcircles of CAD and CDB respectively, such that TH is a common tangent to the two circumcircles and C is inside the quadrilateral $BATH$. Show that $BATH$ is cylic.

G2) Let points P and Q lie on a circle k_1 with centre O. Let k_2 be the circle which is centered at P and passes through Q. Define X as the second intersection of k_2 and line PQ, and Y as the second intersection of k_2 and k_1. Let Z be the intersection of OX with QY. Prove that if $PZYX$ is cyclic, then PYX is an equilateral triangle.

Combinatorics

C1) Let n be a positive integer. Annalena has n different bowls numbered 1 to n and also n apples, $2n$ bananas and $5n$ strawberries. She wants to combine ingredients in each bowl to make fruit salad. The fruit salad is *delicious* if it contains strictly more strawberries than bananas and strictly more bananas than apples. How many ways are there for Annalena to distribute all the fruits to make delicious fruit salad in each of the different bowls?

Remark: A delicious fruit salad is allowed to not contain any apples.

C2) Consider a 2024×2024 grid, where the 2024 squares on one of the two diagonals are coloured blue. Sam writes one of the numbers $1, 2, \ldots, 2024^2$ in each of the squares of the grid in such a way that every number appears exactly once and the squares containing $i-1$ and i share an edge for any $2 \leq i \leq 2024^2$. Prove there are always two blue squares containing values that differ by exactly 2.

Number Theory

N1) Determine all triples (a, b, n) of positive integers where a divides n, b divides n, and

$$(a+1)(b+1) = n.$$

N2) Determine all positive integers n with the property that for each divisor x of n there exists a divisor y of n such that $x + y \mid n$.

Remark: The divisors may be negative.

Good luck!