SMO 1st Round

Lausanne, Zürich - January 14, 2006

Time allowed: 3 hours
Each problem is worth 7 points.

1. Find all triples (p, q, r) of prime numbers such that their pairwise differences

$$|p - q|, \quad |q - r|, \quad |r - p|$$

are also prime numbers.

2. Let n be a positive integer. Determine the number of subsets $A \subset \{1, 2, \ldots, 2n\}$ with the property that there are not two elements $x, y \in A$ with $x + y = 2n + 1$.

3. In triangle ABC define D as the intersection of the angle bisector of $\sphericalangle BAC$ with the side BC. Assume that the circumcenter of triangle ABC coincides with the incenter of triangle ADC. Determine the angles of $\triangle ABC$.

4. Find all positive integer solutions to the equation

$$\operatorname{lcm}(a, b, c) = a + b + c.$$

5. An $m \times n$-board is divided into unit squares. An L-triomino consists of three unit squares: a central square and two outer squares. An L-triomino is located in the upper left corner of the board, the central square covering the corner square of the board. In a move one may rotate the triomino around the midpoint of one of its outer squares by multiples of $90°$. For which m and n is it possible to move the triomino to the lower right corner of the board using a finite number of moves?

Good Luck!