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Final Round 2026

First Exam

Aarburg
March 13, 2026

Duration: 4 hours
Difficulty: The problems are ordered by difficulty.
Points: Each problem is worth 7 points.

1. Let n be a positive integer. Archi and Baldibus are constructing a number on a blackboard, one digit at a time. To start, Archi writes down one of the **nine** digits from 1 to 9. Then, starting with Baldibus, the two players alternate the following:

- Baldibus appends one of the **ten** digits from 0 to 9 to the **end** of the current number.
- Archi prepends one of the **nine** digits from 1 to 9 to the **front** of the current number.

Archi wins if, immediately after Archi writes a digit, the number on the board is divisible by n. Determine the values of n for which Archi can guarantee a win, no matter what Baldibus does.

2. Let $n, k \geq 2$ and $a_1, a_2, \ldots, a_k$ be positive integers satisfying $n < k < 2n$ and

$$2^n - 1 \mid 2^{a_1} + 2^{a_2} + \cdots + 2^{a_k}.$$

Show that at least three of $a_1, a_2, \ldots, a_k$ have the same remainder when divided by n.

3. Let Ω be a circle with diameter $[AB]$ and C a point on the segment $[AB]$. Let ℓ_1 and ℓ_2 be two lines, such that the projections of C onto them both lie on Ω. Let W and X be intersections of the tangent to Ω at A with ℓ_1 and ℓ_2 respectively. Let Y and Z be intersections of the tangent to Ω at B with ℓ_1 and ℓ_2 respectively. Prove that WZ, XY and AB concur at a point.

4. Determine all functions $f\colon \mathbb{R} \to \mathbb{R}$ such that for all $x, y \in \mathbb{R}$,

$$f(x+y) + f(f(y)x + y) = f(x+2y) + f(y)x.$$

Good luck!

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Final Round 2026

Second Exam

Aarburg

March 14, 2026

Duration: 4 hours

Difficulty: The problems are ordered by difficulty.

Points: Each problem is worth 7 points.

5. We call a strictly increasing sequence $a_1, a_2, \ldots, a_{2026}$ of positive integers *indivisible* if for all $2 \leq k \leq 2026$, the sum of any k consecutive terms is **not** divisible by k.

Prove that an indivisible sequence exists and determine the smallest possible value of the last term in an indivisible sequence.

6. Let Γ_1 and Γ_2 be two circles tangent at T, such that Γ_2 is inside of Γ_1. Let PQ be a chord of Γ_1, tangent to Γ_2 at C. Point $A \neq T$ lies on Γ_1 and AT intersects Γ_2 at $B \neq T$. Let line BC intersect AP and AQ at X and Y, respectively. Prove that TB bisects the angle $\angle XTY$.

7. There are finitely many integers on a board, all of which are at least 2. Pingu repeatedly carries out the following step: first, Pingu chooses an integer $n \geq 2$ that is coprime to every integer currently on the board. Pingu then replaces each integer x on the board with the unique integer $y \in \{1, 2, \ldots, n-1\}$ such that $xy \equiv 1 \pmod{n}$.

Pingu stops as soon as there is a 1 anywhere on the board at the end of a step. If at this point, there is also a number different from 1 on the board, Pingu loses. Otherwise, if all the numbers are 1, Pingu wins. Prove that Pingu can always win, regardless of the starting integers.

Example: In a step, if Pingu chooses $n = 11$, Pingu replaces $3, 8, 13, 8$ with $4, 7, 6, 7$.

8. There are 2026 white balls and 2026 black balls arranged in a row in some order. A *balanced segment* is a non-empty segment of consecutive balls that contains the same number of white balls as black balls. A *balanced removal* is the operation of selecting a balanced segment S, removing all the balls in S, and shifting left the balls that were to the right of S so that the remaining balls again form a contiguous row.

Determine the smallest positive integer N satisfying the following property: for every starting configuration of balls and for every integer k satisfying $0 \leq k \leq 2026$, there exists a sequence of at most N balanced removals after which there are exactly $2k$ remaining balls.

Good luck!